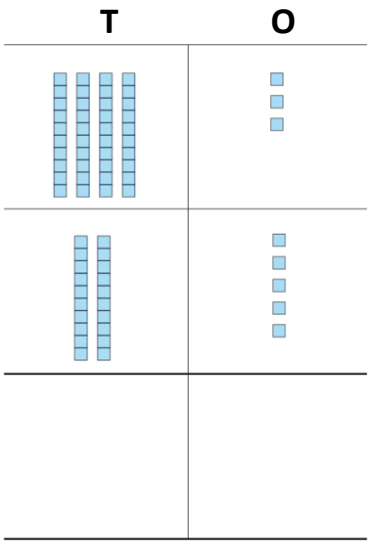
Southdale
C of E Junior school
Shining like stars in the universe

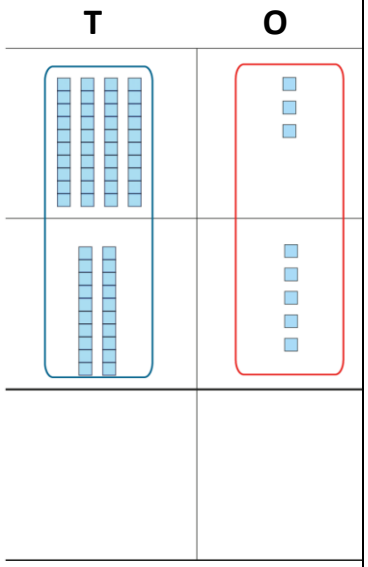
Addition

2 digit plus 2 digit
Use place value headers and diennes to represent the calculation



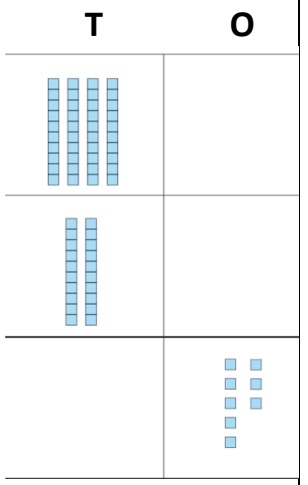
	T	O
	4	3
+	2	5

Model: *We add the ones.3 ones plus 5 ones is equal to 8 ones.*

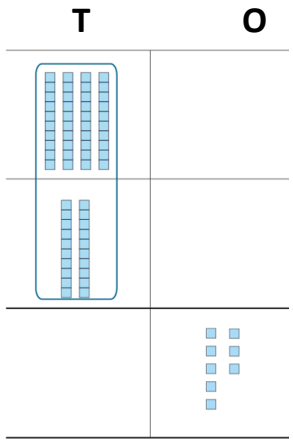
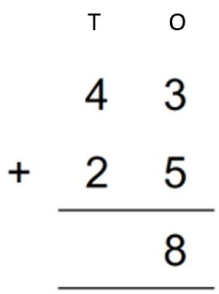
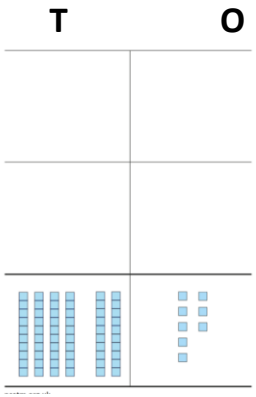
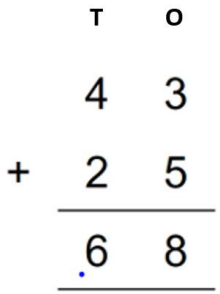
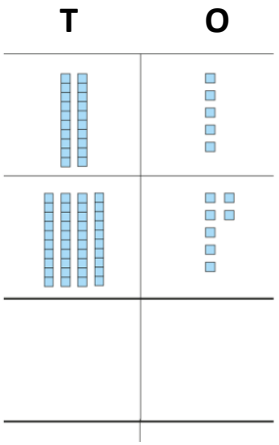
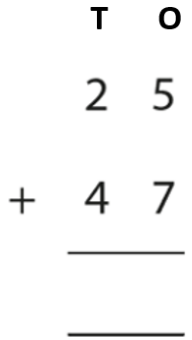
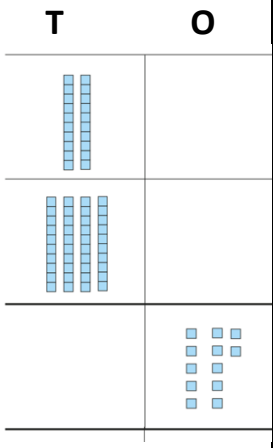
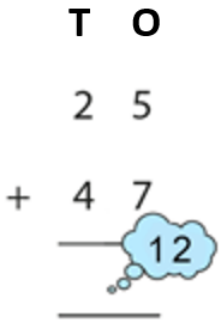
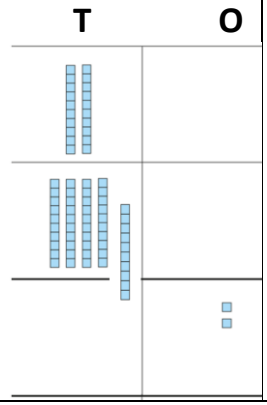
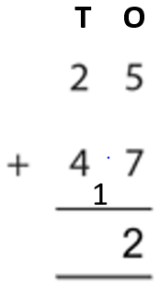


	T	O
	4	3
+	2	5

Move the ones down *3 ones plus 5 ones is equal to 8 ones.*

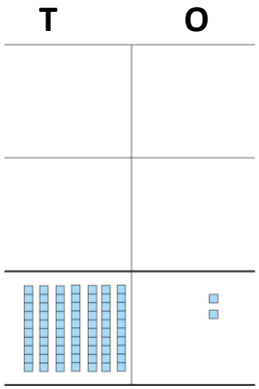


	T	
	4	3
+	2	5

Repeat for the tens *We add the tens. 4 tens plus 2 tens is equal to 6 tens.* 		Move the tens down *4 tens plus 2 tens is equal to 6 tens.* 		Model using the visuliser Model on the maths wall Model using visuals on IWB Always use place value headers and write the calculation alongside.
With regrouping/exchanging 		Bring the ones down 	5 ones plus 7 ones is equal to 12 ones. I can regroup 12 ones. 12 ones is equal to 1 ten and 2 ones. 	5 ones plus 7 ones is equal to 12 ones. I can regroup 12 ones. 12 ones is equal to 1 ten and 2 ones. 	12 ones is equal to 1 ten and 2 ones. 

Add the tens

2 tens plus 4 tens is 6 tens. We also need to add 1 ten from the regrouping. There are 7 tens altogether.



	T	O
	2	5
+	4	7
	1	
	7	2

<u>If a column group is equal to ten or more we must regroup.</u>

<u>10 ones is equivalent to 1 ten. 10 tens is equivalent to 1 hundred etc.</u>

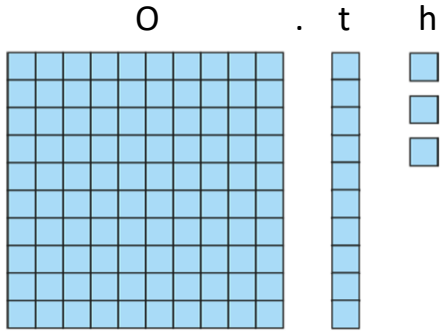
Represent up to one thousand – use through Y3 and the start of Y4

Principle for all calculations

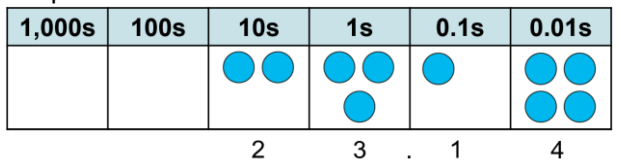
NEVER leave a space for the decimal point

For decimals (Y5/6)

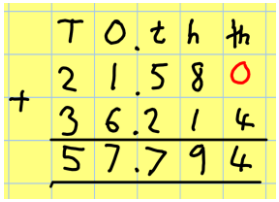
Diennes represent 1.13



For decimals – use place value headers –after using diennes, counters can be used as a representation.



Place holders are used where there is a zero value e.g.



Subtraction – Before beginning on column subtraction, it is important to ensure that children have already mastered representing subtraction with horizontal expressions and that they are confident in using a range of mental strategies for subtraction. ‘

$$65 - 23 = 65 - 20 - 3$$
$$= 45 - 3$$
$$= 42$$

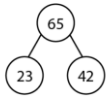
$$\mathbf{65 - 23 = 42}$$

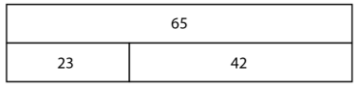
Ensure that children are confident in using the generalised statement:

‘Minuend minus subtrahend is equal to the difference.’

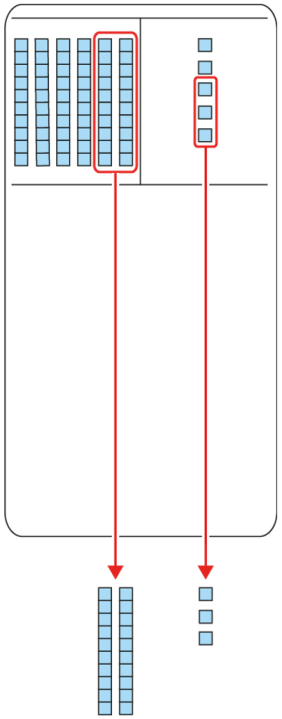
Use a range of familiar layouts alongside teaching the column method so children see the relationship between the numbers.

Part–part–wholes:





No exchanges
2 digit subtract 2 digit - Use place value headers and diennes to represent the calculation. No exchanges. Write the calculation alongside.



Five ones minus three ones is equal to two ones.

Six tens minus two tens is equal to four tens.

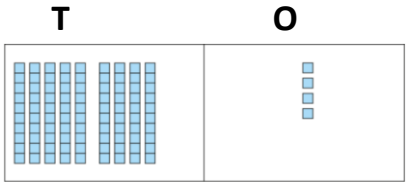
Keep the subtrahend visible to teach inverse operations

	T	O
	6	5
−	2	3
	4	2

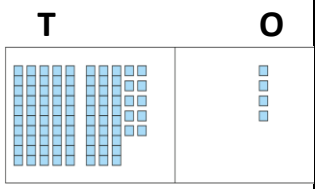
The ones column represents five ones minus three ones is equal to two ones.
The tens column represents six tens minus two tens is equal to four tens.

Model using the visuliser
Model on the maths wall
Model using visuals on IWB
Always use place value headers and write the calculation alongside.

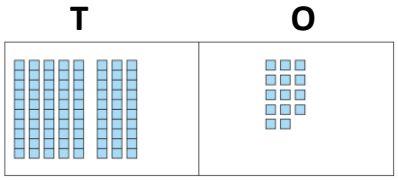
With exchanging
94 – 6
Show the manipulatives first.



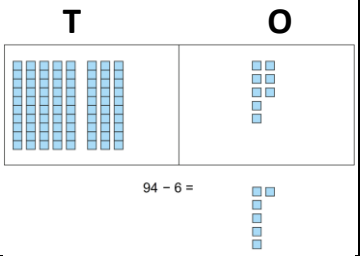
Exchange one ten for ten ones



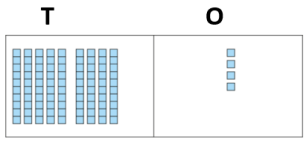
Move the ones to the ones column



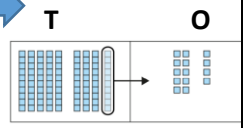
Move the six ones down



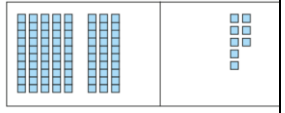
Now introduce the abstract calculation alongside
94 – 6



	T	O
	9	4
–		6



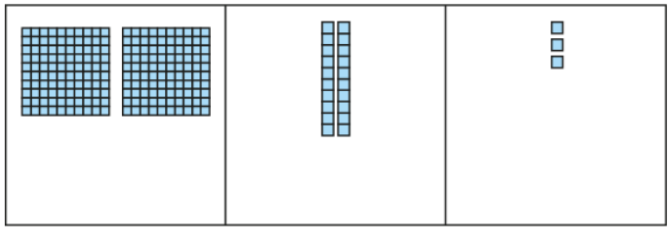
	T	O
	~~9~~ 8	14
–		6



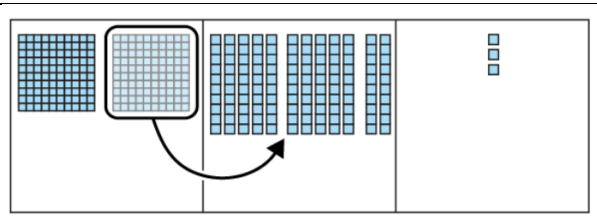
	T	O
	~~9~~ 8	14
–		6
	8	8

Repeat with 3 digit calculations

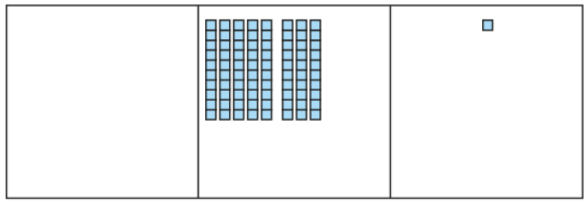
For decimals – see addition section



	100s	10s	1s
	2	2	3
–	1	4	2



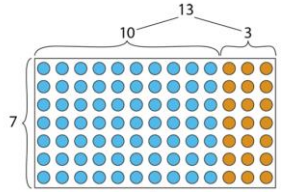
	100s	10s	1s
	~~2~~ 1	12	3
−	1	4	2

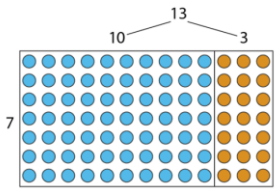


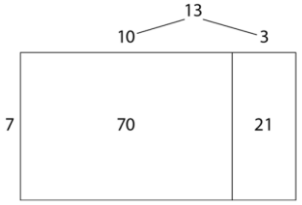
	100s	10s	1s
	~~2~~ 1	12	3
−	1	4	2
	0	8	1

Multiplication

Before moving on to formal methods, children need to be secure in their understanding of the distributive law and multiplying by ten.







$13 \times 7 = 10 \times 7 + 3 \times 7$

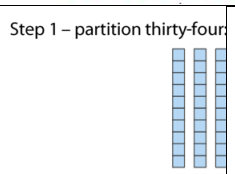
$= 70 + 21$

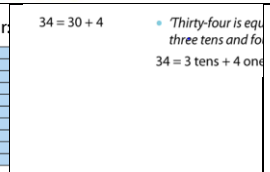
$= 91$

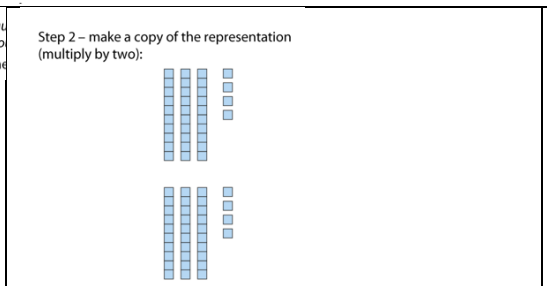
$7 \times 13 = 7 \times 10 + 7 \times 3$

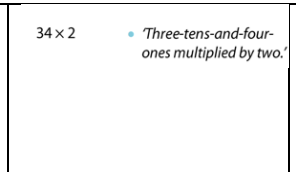
$= 70 + 21$

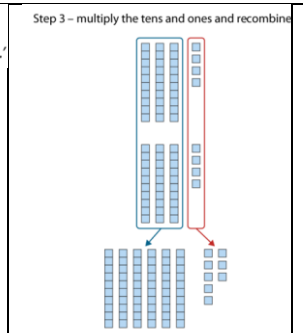
$= 91$

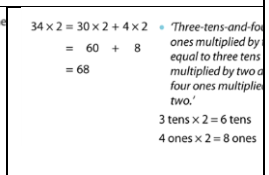












Step 1 – write the factors:

	10s	1s
	3	4
×		2

Step 2 – multiply the single-digit number by the ones

	10s	1s
	3	4
×		2
		8

2 × 4 ones = 8 ones

Step 3 – multiply the single-digit number by the tens:

	10s	1s
	3	4
×		2
		8
	6	0

2 × 4 ones = 8 ones

2 × 3 tens = 6 tens

Step 4 – add the partial products:

	10s	1s
	3	4
×		2
		8
	6	0
	6	8

2 × 4 ones = 8 ones

2 × 3 tens = 6 tens

Informal written method:

$34 \times 2 = 30 \times 2 + 4 \times 2$

$= 60 + 8$

$= 68$

Expanded multiplication algorithm:

	10s	1s	
	3	4	
×		2	
		8	2 × 4 ones = 8 ones
	6	0	2 × 3 tens = 6 tens
	6	8	

Compact layout

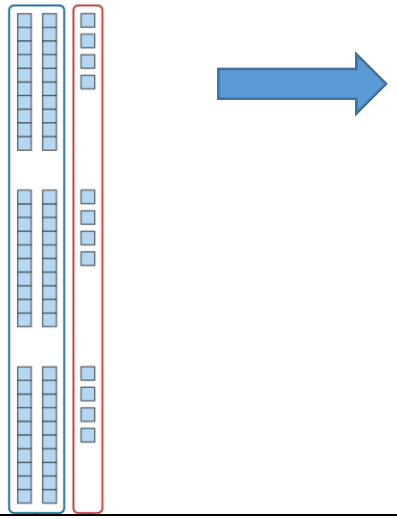
	10s	1s
	3	2
×		3
	9	6

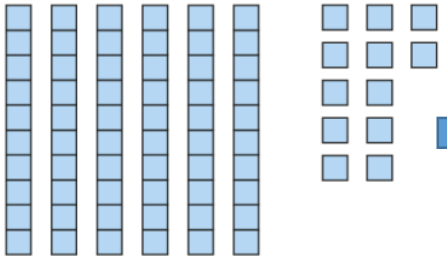
- 3 × 2 ones = 6 ones
  'Write "6" in the ones column.'
- 3 × 3 tens = 9 tens
  'Write "9" in the tens column.'

With regrouping Show with diennes first

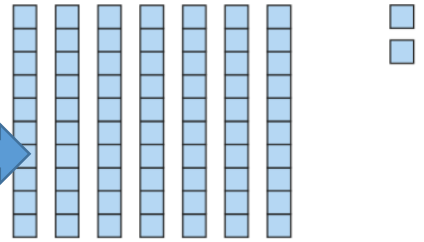
3 x 24

Dienes representation:





There are 6 tens and 12 ones.



There are 6 tens and 12 ones.
The ones can be regrouped as 1 ten and two ones.

There are 7 tens and 2 ones.

Multiplication algorithm – expanded layout:

	10s	1s	
	2	4	
×		3	
	1	2	3 × 4 ones = 12 ones = 1 ten + 2 ones
	6	0	3 × 2 tens = 6 tens
	7	2	

Secure method 24 x 3

Step 2 – write the factors:

	10s	1s
	2	4
×		3

Step 2 – multiply the single digit number by the ones and regroup:

	10s	1s
	2	4
×		3
	1	
		2

3 x 4 ones = 12 ones

= 1 ten + 2 ones

Write one below the tens column and 2 in the ones column.

Step 3 – multiply the single digit by the tens and add the tens from regrouping.

	10s	1s
	2	4
×		3
	1	
	7	2

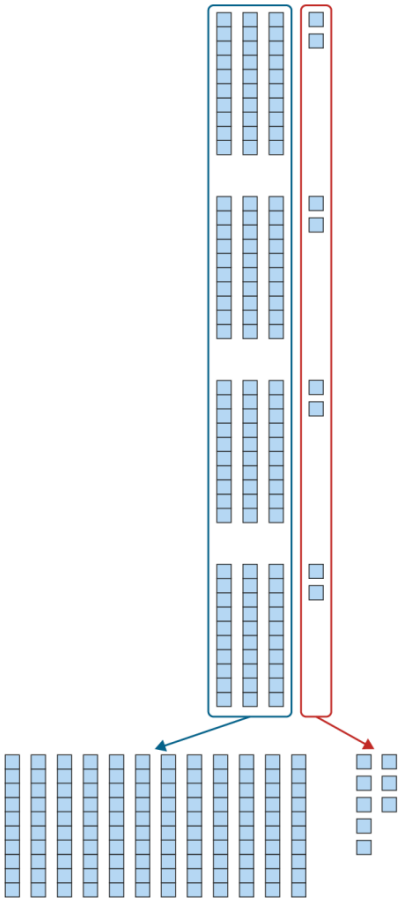
3 x 2 tens = 6 tens

6 tens + 1 ten = 7 tens

Write the 7 in the tens column

Regrouping of tens into hundreds 32 x 4

Multiply the tens and ones and recombine:



$32 \times 4 = 30 \times 4 + 2 \times 4$

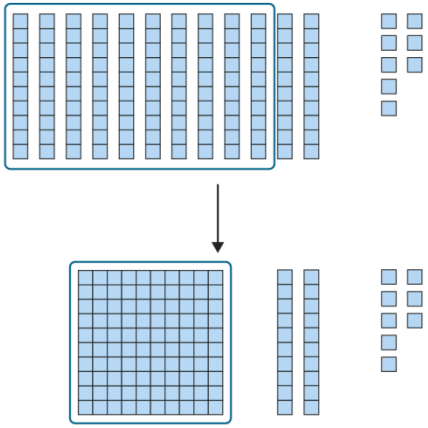
$= \quad 120 \quad + \quad 8$

- *'Three-tens-and-two-ones multiplied by four is equal to three tens multiplied by four and two ones multiplied by four.'*

3 tens × 4 = 12 tens

2 ones × 4 = 8 ones

Regroup the tens into hundreds and tens:



$32 \times 4 = 120 + 8$

$= 128$

- *'Twelve tens is equal to one hundred and two tens.'*

12 tens = 1 hundred + 2 tens

Multiplication algorithm – expanded layout:

	100s	10s	1s
		3	2
×			4
			8
	1	2	0
	1	2	8

- 4 × 2 ones = 8 ones
- 4 × 3 tens = 12 tens = 1 hundred + 2 tens

Example 1 – compact layout *with* place-value headings:

	100s	10s	1s
		3	2
×			4
	1	2	8

- 4 × 2 ones = 8 ones
  'Write "8" in the ones column.'
- 4 × 3 tens = 12 tens = 1 hundred + 2 tens
  'Write "1" in the hundreds column and "2" in the tens column.'

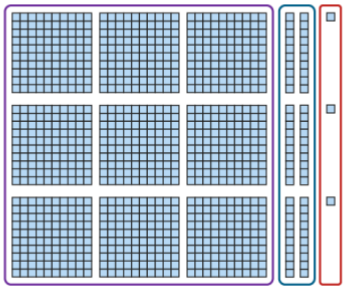
Example 2 – compact layout *without* place-value headings:

```
   2 1
×    5
-------
 1 0 5
```

- 5 × 1 one = 5 ones
  'Write "5" in the ones column.'
- 5 × 2 tens = 10 tens; 10 tens = 1 hundred and 0 te
  'Write "1" in the hundreds column and "0" in the ten column.'

321 × 3

Dienes representation:



Multiplication algorithm – expanded layout:

	100s	10s	1s	
	3	2	1	
×			3	
			3	3 × 1 ones = 3 ones
		6	0	3 × 2 tens = 6 tens
	9	0	0	3 × 3 hundreds = 9 hundreds
	9	6	3	

Multiplication algorithm – compact layout:

```
  3 2 1
×     3
-------
  9 6 3
```

Multiplication algorithm – expanded layout:

	1,000s	100s	10s	1s
		3	6	7
×				4
			2	8
		2	4	0
	1	2	0	0
	1	4	6	8

4×7 ones $= 28$ ones
$= 2$ tens $+ 8$ ones

4×6 tens $= 24$ tens
$= 2$ hundreds $+ 4$ tens

4×3 hundreds $= 12$ hundreds
$= 1$ thousand $+ 2$ hundreds

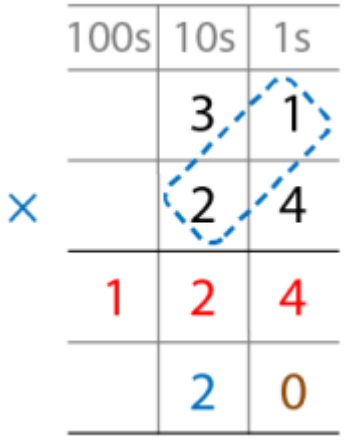
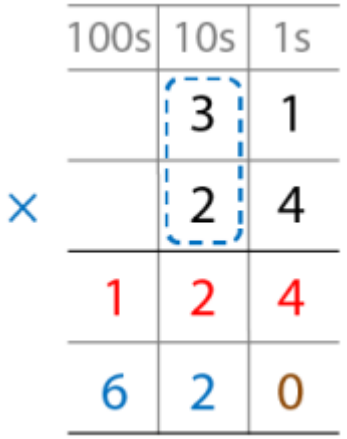
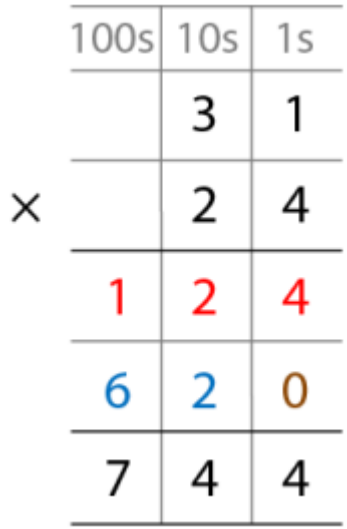
Multiplication algorithm – compact layout:

$$\begin{array}{rcccc} & & 3 & 6 & 7 \\ \times & & & & 4 \\ \hline & & \overset{2}{\cancel{2}} & \overset{2}{\cancel{2}} & \\ & 1 & 4 & 6 & 8 \\ \hline \end{array}$$

- *'First, write the largest factor: "367".'*
- *'Then write the smallest factor below, lining up the digits: "4".'*
- *'Now multiply, starting with the ones: four times seven ones is equal to twenty-eight ones…'*
- *'…and regroup: twenty-eight ones is equal to two tens and eight ones; write "8" in the ones column and "2" below the tens column.'*
- *'Then move to the tens: four times six tens is equal to twenty-four tens…'*
- *'…and regroup: twenty-four tens is equal two hundreds and four tens…'*
- *'…and add the two tens from regrouping to give two hundreds and six tens: write "6" in the tens column and "2" below the hundreds column.'*
- *'Then move to the hundreds: four times three hundreds is equal to twelve hundreds…'*
- *'…and regroup: twelve hundreds is equal to one thousand and two hundreds…'*
- *'…and add the two hundreds from regrouping to give one thousand and four hundreds; write "1" in the thousands column and "4" in the hundreds column.'*

Decimal multiplication $\begin{array}{r} 5.7 \\ \times \quad 3 \\ \hline 17.1 \\ \hline \end{array}$ (2)	$\begin{array}{r} 0.62 \\ \times \quad 8 \\ \hline 4.96 \\ \hline \end{array}$ (1)	$\begin{array}{r} 12.7 \\ \times \quad 6 \\ \hline 76.2 \\ \hline \end{array}$ (1 4)	$\begin{array}{r} 4.56 \\ \times \quad 4 \\ \hline 18.24 \\ \hline \end{array}$ (2 2)

long multiplication expanded layout Step one: write the factors 100s | 10s | 1s .3 | 1 × 2 | 4	Step two: multiply the ones digits by the ones digits 100s | 10s | 1s 3 | 1 × 2 | 4 4	Step three: multiply the tens digits by the ones digit and regroup 100s | 10s | 1s 3 | 1 × 2 | 4 1 | 2 | 4
Step four: Place a zero to show that it is ten times the size	Step five: Multiple the ones digit by the tens digit	Step 6: Multiply the tens digit by the tens digit

		
Step seven: Add the partial products 		

Compact layout 312 x 28 1 2496 6240 1 8736	3126 x 28 124 25008 1 62520 87528	

division

Children are first taught the informal method

84 ÷ 4 = ?

8 tens ÷ 4 = 2 tens

4 ones ÷ 4 = 1 one

8 tens	÷	4	=	2 tens
4 ones	÷	4	=	1 one
84	÷	4	=	21

72 ÷ 3 = ?

7 tens ÷ 3 = 2 tens r 1 ten

1 ten and 2 ones = 12 ones

12 ones ÷ 3 = 4 ones

6 tens	÷	3	=	2 tens
12 ones	÷	3	=	4 ones
72	÷	3	=	24

73 ÷ 3 = ?

7 tens ÷ 3 = 2 tens r 1 ten

1 ten and 3 ones = 13 ones

13 ones ÷ 3 = 4 ones r 1 one

6 tens	÷	3	=	2 tens
13 ones	÷	3	=	4 ones r 1 one
73	÷	3	=	24 r 1

Formal method – taught with pictorial alongside

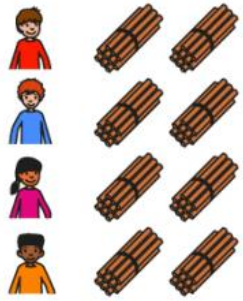
Step 1 – write the divisor and dividend

10s 1s

$$4\overline{)\,8\quad 4}$$

'Eighty-four divided by four.'

Step 2 – sharing the tens



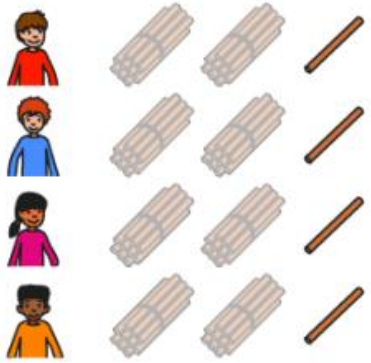
10s 1s

$$\begin{array}{r} 2 \\ \hline 4\overline{)\,8\quad 4} \end{array}$$

8 tens ÷ 4 = 2 tens

'Eight tens divided by four is equal to two tens.'

Step 3 – sharing the ones

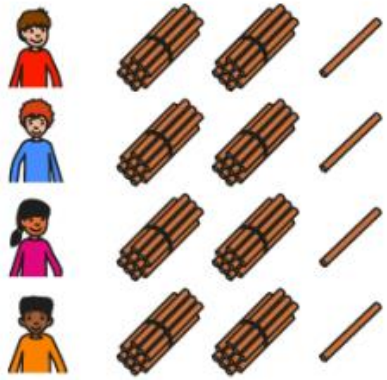


10s	1s
2	1
4) 8	4

8 tens ÷ 4 = 2 tens
4 ones ÷ 4 = 1 one

'Four ones divided by four is equal to one one.'

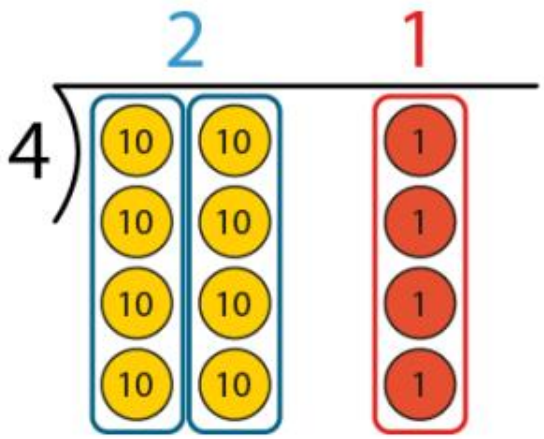
Summary



10s	1s
2	1
4) 8	4

'Each child gets twenty-one sticks.'

Algorithm with place value counters summary



8 tens ÷ 4 = 2 tens

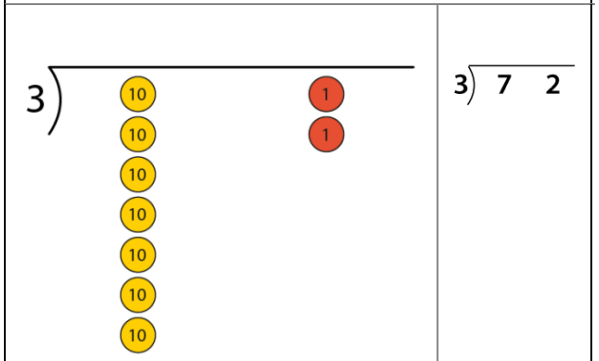
4 ones ÷ 4 = 1 one

- *'Eight tens and four ones divided between four is equal to two tens and one one.'*
- *'Each child gets twenty-one sticks.'*

84 ÷ 4 = 21	$4\overline{)84}$ (2 1)
dividend ÷ divisor = quotient	$\text{divisor}\overline{)\text{dividend}}$ (quotient)

With exchanges

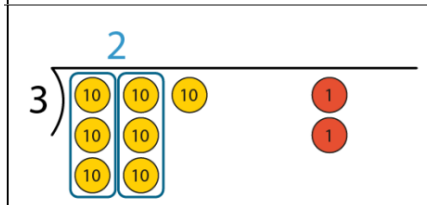
Step 1 – write the divisor and dividend



3) 7 2

'Seventy-two divided by three.'

Step 2 – sharing the tens…

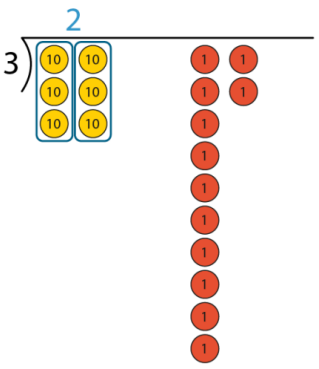


2
3) 7 2

7 tens ÷ 3 = 2 tens r 1 ten

'Write "2" in the tens column…'

Step 3 – …and exchanging



2
3) 7 ^{1}2

1 ten = 10 ones

'…and write "1" to the left of the ones digit of the dividend to make twelve ones.'

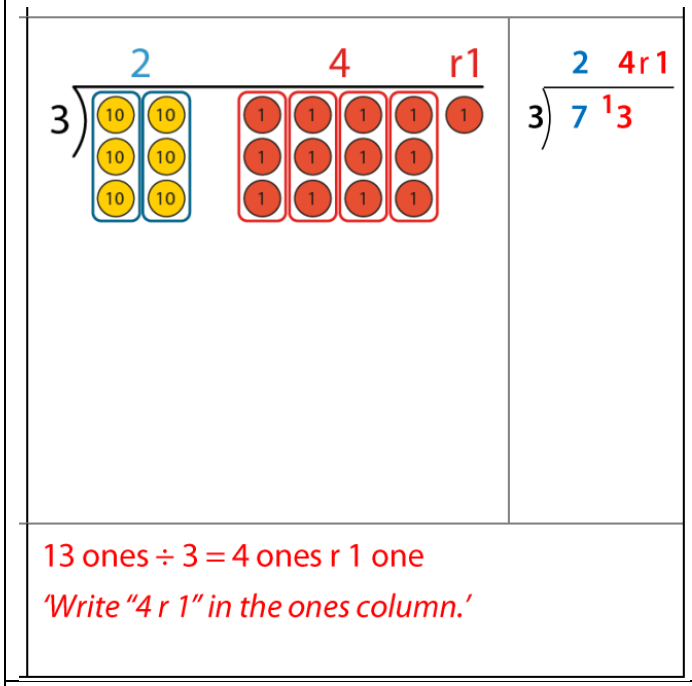
Step 4 – sharing the ones

2 4
3) 7 ^{1}2

12 ones ÷ 3 = 4 ones

'Write "4" in the ones column.'

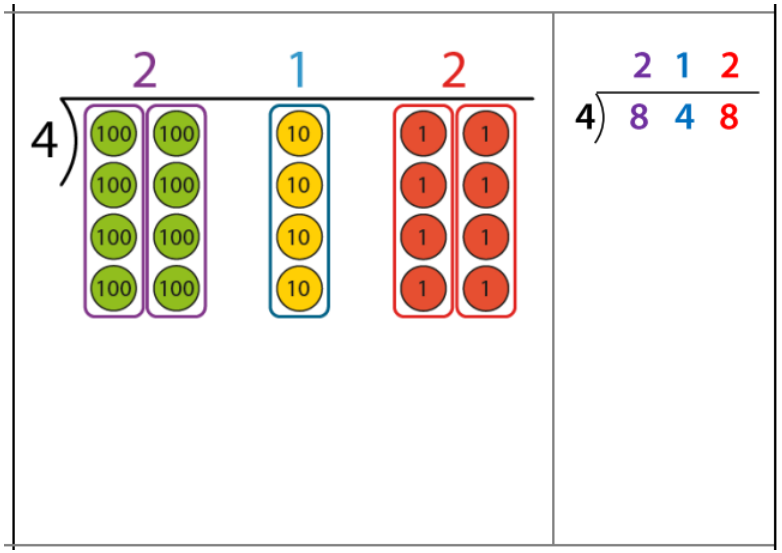
With remainders



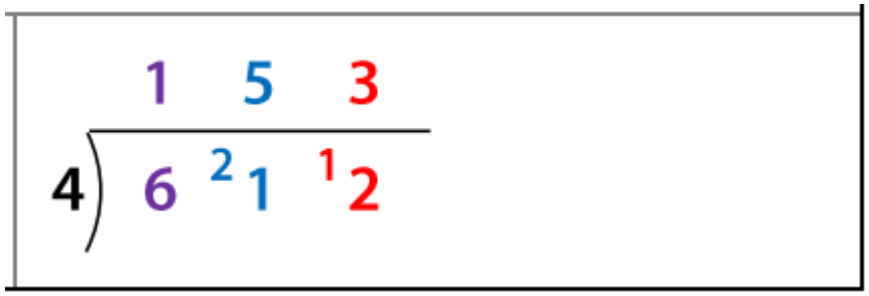
13 ones ÷ 3 = 4 ones r 1 one

'Write "4 r 1" in the ones column.'

With hundreds



Short method



Long division

$$\begin{array}{r} \,9\;\;r\,7 \\ 32\overline{)\,2\;\;9\;\;5} \\ \underline{\;2\;\;8\;\;8\;\;} \\ \,0\;\;0\;\;7 \end{array} \qquad (9\times 32)$$

so

$295 \div 32 = 9 \text{ r } 7$

431 divided by 31

Step 1 – write the divisor, frame and dividend

$$31\overline{)\,4\;\;3\;\;4}$$

Step 2 – divide the hundreds

$$\begin{array}{r} \,0 \\ 31\overline{)\,4\;\;3\;\;4} \end{array}$$

4 hundreds ÷ 31 = 0 hundreds r 4 hundreds

- *'Write "0" in the hundreds column of the answer line.'*

Step 3 – exchange hundreds for tens, combine with the existing tens and divide…

$$\begin{array}{r} 0\ 1 \\ 31\overline{)4\ 3\ 4} \\ 3\ 1 \end{array}$$

(1 ten × 31 = 31 tens)

4 hundreds = 40 tens

40 tens + 3 tens = 43 tens

43 tens ÷ 31 = 1 ten and a remainder

- *'Write "1" in the tens column of the answer line and write "31" underneath the "43".'*

Step 4 – subtract to find the remainder

$$\begin{array}{r} 0\ 1 \\ 31\overline{)4\ 3\ 4} \\ 3\ 1 \\ \hline 1\ 2 \end{array}$$

(1 ten × 31 = 31 tens)

43 tens – 31 tens = 12 tens

- *'Write "12" underneath the "31".'*

Step 5 – exchange tens for ones and combine with the existing ones

$$\begin{array}{r} 0\ 1 \\ 31\overline{)4\ 3\ 4} \\ 3\ 1\ \downarrow \\ \hline 1\ 2\ 4 \end{array}$$

(1 ten × 31 = 31 tens)

12 tens = 120 ones

120 ones + 4 ones = 124 ones

- *'Write "4" after the "12".'*

Step 6 – divide the ones

$$\begin{array}{r} 0\ 1\ 4 \\ 31\overline{)4\ 3\ 4} \\ 3\ 1 \\ \hline 1\ 2\ 4 \\ 1\ 2\ 4 \end{array}$$

(1 ten × 31 = 31 tens)

(4 ones × 31 = 124 ones)

124 ones ÷ 31 = 4 ones

(refer to the ratio chart)

- *'Write "4" in the ones column of the answer line and write "124" underneath the "124", aligning the digits.'*

Step 7 – subtract to show there is no remainder

$$\begin{array}{r} 0\ 1\ 4 \\ 31\overline{)\,4\ 3\ 4} \\ \underline{3\ 1\ \ } \\ 1\ 2\ 4 \\ \underline{1\ 2\ 4} \\ 0 \end{array}$$

(1 ten × 31 = 31 tens)

(4 ones × 31 = 124 ones)

124 ones – 124 ones = 0 ones

- *'Write "0" underneath the "31".'*

$$\begin{array}{r} 2\ 1\ 5 \\ 23\overline{)\,4\ 9\ 4\ 5} \\ \underline{4\ 6\ \ \ } \\ 3\ 4 \\ \underline{2\ 3\ \ } \\ 1\ 1\ 5 \\ \underline{1\ 1\ 5} \\ 0 \end{array}$$

Long division – remainder converted to a decimal

$$\begin{array}{r} 2\ 9.2 \\ 25\overline{)\,7\ 3\ 0.0} \\ \underline{5\ 0\ \ } \\ 2\ 3\ 0 \\ \underline{2\ 2\ 5} \\ 5\ 0 \\ \underline{5\ 0} \\ 0 \end{array}$$

- 50 tenths ÷ 25 = 2 tenths
  'Write "2" in the tenths column of the answer line and write "50" underneath the "50".'
- 50 tenths – 50 tenths = 0 tenths
  'Write "0" underneath the "50".'

Long division remainder

$$\begin{array}{r} 23\ r9 \\ 15\overline{)354} \\ 30 \\ \hline 54 \\ 45 \\ \hline 9 \end{array}$$

So, $354 \div 15 = 23$ r 9

Long division – proper fraction

$$\begin{array}{r} 23\ \frac{9}{15} \\ 15\overline{)354} \\ 30 \\ \hline 54 \\ 45 \\ \hline 9 \end{array}$$

$\frac{9}{15} = \frac{3}{5}$

So, $354 \div 15 = 23\frac{3}{5}$

Long division – decimal

$$\begin{array}{r} 23.6 \\ 15\overline{)354.0} \\ 30 \\ \hline 54 \\ 45 \\ \hline 90 \\ 90 \\ \hline 0 \end{array}$$

So, $354 \div 15 = 23.6$